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# All or nothing: State capacity and optimal public goods provision ☆

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**Abstract**

We study public goods provision subject to ex post incentive and participation constraints. We also impose a requirement of anonymity. Different public goods can be bundled if sufficient resources are available. The analysis focuses on the *all-or-nothing-mechanism*: Expand provision as much as is resource feasible if no one vetoes - otherwise stick to the status quo. We show that the probability of the all-outcome converges to one as the capacity becomes unbounded. For a given finite capacity, we provide conditions under which the all-or-nothing-mechanism is ex ante welfare-maximizing - even though, ex post, it involves an overprovision of public goods.


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## 1. Introduction

We study the following situation: There is a status quo with a limited provision of public goods. Moving towards more goods being provided requires both sufficient resources and sufficient political support. Our main result shows that an increase in capacity, i.e. in resources available to finance public goods, makes it possible to overcome all obstacles to increased public goods provision. It eliminates resistance by those who dislike certain public goods and it eliminates incentives to free-ride on the contributions of others. We also provide conditions under which this mechanism maximizes expected welfare.

The paper contributes to the literature that studies public goods provision from a mechanism design perspective. By and large, the existing literature, reviewed in more detail below, emphasizes the difficulties that are associated with incentive and participation constraints. The second-best mechanisms that respect these constraints typically involve an underprovision of public goods. By contrast, our setting – which, in addition, invokes a requirement of anonymity – gives rise to a second-best mechanism with an overprovision of public goods.

A seminal reference is Mailath and Postlewaite (1990) who show that, as the number of individuals gets large, the probability of public good provision goes to zero under any mechanism that respects incentive and participation constraints. A tempting conclusion therefore is that a requirement of unanimity in favor of increased public good provision makes it impossible to have significant expenditures on public goods. Such expenditures can then be reconciled only with a violation of participation constraints or, equivalently, a use of the government's coercive power to finance public goods, against the will of at least some of the people. Against this background, our analysis shows a possibility to have substantial public goods provision in the presence of participation constraints: An increasing capacity allows to bundle public goods in such a way that moving towards increased expenditures is in everyone's interest.

**Sketch of the formal analysis.** There are $n$ individuals and there is sufficient capacity to finance $m$ additional public goods. Individuals have private information on their valuations of these goods. For anyone else, valuations are taken to be $iid$ random variables with a mean that exceeds the per capita provision cost and which take values lower than the cost with positive probability. Thus, it is a priori unclear which public goods should be provided.

A mechanism determines which goods are provided and also what individuals have to pay. Admissible mechanisms satisfy participation, incentive and budget constraints. We require that all these constraints hold ex post. Thus, whatever the state of the economy, ex post, no individual prefers the status quo over the outcome of the mechanism, nor does any one individual regret to have revealed her preferences. In addition, the money that is collected from individuals is exactly what is needed to cover the cost of provision. We also impose a condition of anonymity.

Mailath and Postlewaite (1990) have established an impossibility result for the case $m = 1$: With many individuals, the probability of public goods provision is close to zero under any admissible mechanism. Mailath and Postlewaite employ participation, incentive and resource constraints that are more permissive than ours. In their analysis, participation constraints are satisfied if all individuals' expected utility under the mechanism is higher than in the status quo. Incentive compatibility holds if a truthful revelation of preferences is a Bayes-Nash equilibrium, rather than an ex post or dominant strategy equilibrium. Our analysis shows that the impossibility of public goods provision can be overcome if many public goods are provided simultaneously. An impossibility result in mechanism design gets stronger with weaker constraints. A possibility result gets stronger with stronger constraints. Thus, while for the purposes of Mailath and Postlewaite, it was a natural choice to have constraints that need to hold only in expectation, for us, the natural choice is to have separate participation, incentive and budget constraints for each state of the economy.[1]

The all-or-nothing-mechanism plays a decisive role in our analysis. This mechanism has only two outcomes: Either the status quo prevails, or the capacity for increased public goods provision is exhausted. Costs are shared equally among individuals. Exhausting the capacity requires a consensus. As soon as one individual opts for the status quo, the status quo stays in place. This mechanism is obviously admissible: The veto rights ensure that participation constraints are satisfied. If no one makes use of his veto power, then, whatever the preference profile, the mechanism stipulates the same outcome. This limited use of information on preferences ensures incentive compatibility.

Our first set of results shows that, under the all-or-nothing-mechanism, the probability of the "all-outcome" is an increasing function of the capacity $m$ and converges to 1 as $m$ becomes unbounded. This can be understood as a large numbers effect. The larger the bundle, the closer are individual preferences to the mean of the distribution from which preferences are drawn. As the mean exceeds the per capita cost, the larger the bundle the less likely is a veto. To relate our analysis to Mailath and Postlewaite (1990) we also consider the possibility that both the capacity $m$ and the number of individuals $n$ grow. If this process is such that the ratio $\frac{m}{n}$ converges to a positive constant, the limit probability of the all-outcome is bounded away from zero.

A second set of results establishes conditions under which the all-or-nothing-mechanism is a second-best mechanism, i.e. a mechanism that maximizes the expected surplus over the set of admissible mechanisms. The all-or-nothing-mechanism may not appear as a natural candidate for an optimal mechanism: The all-outcome gives rise to an overprovision of public goods as it typically includes public goods with negative surplus. Since the mechanism offers only the alternatives "all" and "nothing", there is no possibility to eliminate those goods from the bundle. When the nothing-outcome prevails there is an underprovision of public goods. For a given finite capacity, this happens with positive probability. By our first set of results, the probability of underprovision decreases and the probability of overprovision increases in the capacity for public goods provision.

The requirement of anonymity plays a prominent role in this part of our analysis. It is a requirement of equal treatment: A permutation of individual preferences must not affect public goods provision levels. Moreover, the payments of individuals with the same public goods preferences are equal. We show that – in the presence of incentive, participation and budget constraints – there is only one payment rule that is anonymous: equal cost sharing. This finding facilitates the analysis. It makes it possible to focus on the second-best provision rule for public goods – as opposed to having a joint analysis of payment and provision rules.[2]

Our analysis invokes the famous impossibility result by Gibbard (1973) and Satterthwaite (1975). According to this result, with an unrestricted preference domain, any mechanism that is ex post incentive compatible and allows for more than two outcomes is dictatorial. We show that, under an ancillary assumption, this theorem applies to our setup. The implication is that the set of admissible mechanisms becomes small: There can be at most two outcomes. One of the two

[1] Ex post constraints are attractive also for another reason. Mechanisms that satisfy these constraints are robust in the sense that they reach the intended outcome whatever the individuals' probabilistic beliefs about the types of other individuals, see Bergemann and Morris (2005).

[2] For the case $m = 1$, Kuzmics and Steg (2017) characterize all payment rules that satisfy budget balance and dominant strategy incentive compatibility, i.e. including the non-anonymous ones.

outcomes has to be the status quo. Otherwise, it would impossible to respect participation constraints. Thus, the only degree of freedom is the choice of the second outcome. The assumption that public goods provision is desirable in expectation, implies that it is desirable to exhaust the capacity to provide public goods. Thus, a second best mechanism gives a choice between two outcomes, “all” or “nothing”.

The ancillary assumption is that the different public projects are ordered. The project with index $k$ can be implemented only if all projects with an index smaller than $k$ have been implemented already. As we will discuss, this assumption makes the formal analysis tractable. It also has an empirical plausibility for specific types of infrastructure investment. The development of transportation networks (railroads, highways) frequently follows a sequential logic where the major centers are connected before the network is extended towards less important cities, and finally supplemented with branch lines. Economic historians have documented that developments of railway or highway networks in the 19th and early 20th century followed this pattern, see Fogel (1962), Voigtlaender and Voth (2014), Hornung (2015), or Donaldson (2018) for examples.

**Related literature.** The observation that bundling can alleviate inefficiencies due to incentive or participation constraints is due to Jackson and Sonnenschein (2007) and Fang and Norman (2006). Both papers focus on Bayes-Nash equilibria and on participation constraints that need to hold at the interim stage where individuals know their own type but still face uncertainty about the types of others and hence about the outcome of the mechanism. Moreover, both papers show that bundling a large number of decisions allows to approximate first-best outcomes. Our work differs in that we invoke ex post incentive and participation constraints. As a consequence, first-best outcomes cannot be reached. The second-best outcome is the all-or-nothing-mechanism that gives rise to an overprovision of public goods.

If bundling is not an option, second-best mechanisms give rise to an underprovision of public goods.[3] More specifically, Güth and Hellwig (1986) show that the second-best mechanisms involve underprovision. Mailath and Postlewaite (1990) show that, under any admissible mechanism, the probability of public goods provision goes to zero as the number of individuals becomes unbounded. An important assumption is that the per capita cost of provision remains constant as additional individuals are added to the system. Hellwig (2003), by contrast, allows for scale economies. Welfare-maximizing provision levels then increase with the number of individuals. Still, these second-best provision levels may fall short of first-best levels. For excludable public goods, as shown by Norman (2004), second-best mechanisms involve use restrictions to mitigate the distortions from incentive and participation constraints, again with the implication that second-best provision levels are smaller than first-best levels.

**Outlook.** The following section introduces the formal framework. In Section 3, we show that, under the all-or-nothing-mechanism, public expenditures increase in the capacity to provide public goods. Section 4 shows that the all-or-nothing-mechanism is a second-best mechanism. The last section contains concluding remarks. Formal proofs are relegated to the Appendix.

[3] Some qualifications are in order. With correlated, rather than independent types first-best outcomes can typically be reached in the presence of incentive and participation constraints, see Crémer and McLean (1988). With independent types, and without participation constraints, first best outcomes can typically be implemented as a Bayes-Nash equilibrium, see d'Aspremont and Gérard-Varet (1979), but not as a dominant strategy equilibrium, see Green and Laffont (1977).

## 2. The model

The set of individuals is denoted by $I = \{1, \ldots, n\}$. A finite set $K = \{1, \ldots, m\}$ of public projects is available. We interpret $m$ as the measure of capacity; that is, the economy has the resources to finance at most $m$ projects. A mechanism determines which elements of $K$ are implemented and how the costs are shared.

The benefit that individual $i$ realizes if project $k$ is undertaken is denoted by $\theta_{ik}$. We write $\theta_i = (\theta_{ik})_{k \in K}$ for the preference profile of $i$ and denote the set of possible profiles by $\Theta_i$. We write $\theta = (\theta_1, \ldots, \theta_n)$, refer to $\theta$ as a *state of the economy* and to $\Theta = \Pi_{i=1}^{n} \Theta_i$ as the set of states. The values $\theta_{ik}$ are drawn independently from the same distribution $F$ with density $f$. We denote the mean of these random variables by $\mu$ and the variance by $\sigma^2$. Each agent privately observes $\theta_i$.

Without loss of generality, we set the per capita cost of providing any one public project $k$ equal to 1. We denote by $s_k(\theta) = \frac{1}{n} \sum_{i=1}^{n} \theta_{ik} - 1$ the per capita surplus that would be generated if public good $k$ was implemented in state $\theta$. We assume that $\mu > 1$, with the implication that the expected value of $s_k(\theta)$ is positive. We also assume that realizations of $\theta_{ik}$ strictly smaller than 1 occur with positive probability. Hence, negative values of $s_k(\theta)$ have positive probability.

The revelation principle applies so that we can focus on direct mechanisms. A direct mechanism is a collection of functions $q_k : \Theta \to \{0, 1\}$, $k \in K$, that indicate, for each state of the economy, whether public good $k$ is provided or not. In addition, there is a collection of functions $t_i : \Theta \to \mathbb{R}$, $i \in I$, that specify individual payments as a function of the state of the economy. Under such a mechanism, the payoff of individual $i$ in state $\theta$ is given by

$$u_i(\theta) = \sum_{k \in K} \theta_{ik}\, q_k(\theta) - t_i(\theta) \, .$$

We say that a direct mechanism is admissible if it satisfies incentive, participation and budget constraints. Participation constraints hold in an ex post sense if, for all $i$ and $\theta$,

$$u_i(\theta) \geq 0 \, . \tag{1}$$

Incentive compatibility holds provided that truth-telling is an ex post or dominant strategy equilibrium, i.e. if for all $i$, all $\theta = (\theta_i, \theta_{-i})$ and all $\hat{\theta}_i$,[4]

$$u_i(\theta_i, \theta_{-i}) \geq u_i(\hat{\theta}_i, \theta_{-i}) \, . \tag{2}$$

Budget balance requires that, for all $\theta$,

$$\frac{1}{n} \sum_{i=1}^{n} t_i(\theta) = \sum_{k=1}^{m} q_k(\theta) \, . \tag{3}$$

Finally, we require a mechanism to be anonymous, i.e. a permutation of individual types must not affect the outcome of the mechanism. More formally, for any state $\theta = (\theta_1, \ldots, \theta_n)$ and any bijection $\pi : I \to I$, $q_k(\theta_1, \ldots, \theta_n) = q_k(\theta_{\pi(1)}, \ldots, \theta_{\pi(n)})$, for all $k$, and $t_i(\theta_1, \ldots, \theta_n) = t_{\pi(i)}(\theta_{\pi(1)}, \ldots, \theta_{\pi(n)})$, for all $i$.

**The all-or-nothing-mechanism.** The all-or-nothing-mechanism is an admissible mechanism. Under this mechanism, all public goods are provided and the costs are shared equally unless

[4] In environments with private values, ex post and dominant strategy equilibria coincide, see e.g. Bergemann and Morris (2005).

there is an individual who prefers the status quo. In this case, the status quo prevails. Formally: If $\frac{1}{m}\sum_{k=1}^{m}\theta_{jk} < 1$ for some $j \in I$, then $q_k(\theta) = 0$, for all $k$, and $t_i(\theta) = 0$, for all $i$. Otherwise, $q_k(\theta) = 1$, for all $k$, and $t_i(\theta) = m$, for all $i$.

## 3. Capacity and expenditures on public goods

Let $\mathbb{P}_{all}(m)$ be the probability of the all-outcome under an all-or-noting mechanism with capacity $m$. We will use a result from statistics to show that, under a monotone hazard rate assumption, $\mathbb{P}_{all}$ is an increasing function. Thus, the probability of provision is an increasing function of the capacity to provide public goods. We also provide limit results for the case that $m$ becomes unbounded. The limit results hold irrespectively of whether or not the monotone hazard rate assumption is satisfied.

**Proposition 1.** *Suppose $f$ is symmetric around its mean, log-concave and has full support on an interval $[\mu - a, \mu + a]$, for some $a > 0$. Then $\mathbb{P}_{all}(m)$ increases monotonically in $m$.*

The result of Mailath and Postlewaite (1990) applies to the case $m = 1$: $\mathbb{P}_{all}(1)$ is close to zero if the number of individuals $n$ is sufficiently large.[5] If the density $f$ is both symmetric and log-concave, then the probability of the all-outcome is larger if the capacity suffices to finance two public projects, $\mathbb{P}_{all}(2) > \mathbb{P}_{all}(1)$ and even larger if it suffices to finance three public projects and so on.[6] According to Proposition 2 this sequence of probabilities converges to 1, i.e. as $m$ grows without bound the probability that there is an individual who prefers the status quo over the all-outcome vanishes.

The assumption of symmetry is needed for Proposition 1. Without this assumption, one can show that, for any $m$, there exists some $m' > m$ so that $\mathbb{P}_{all}(m') > \mathbb{P}_{all}(m)$. It is, however, not possible to exclude the possibility that $\mathbb{P}_{all}(m + 1) < \mathbb{P}_{all}(m)$ for some $m$. Proposition 2, by contrast, holds irrespectively of whether or not $f$ is symmetric.

**Proposition 2.** $\lim_{m\to\infty} \mathbb{P}_{all}(m) = 1$.

The proposition follows from a straightforward application of Chebychev's inequality. Intuitively, as $m$ grows without bound, for any individual $i$, $\frac{1}{m}\sum_{k=1}^{m}\theta_{ik}$ converges to $\mu$ by a large numbers effect. Providing all public goods is therefore in everyone's interest.

Suppose that $m = 1$ and that $n$ is large. The per capita valuation of the public good $\frac{1}{n}\sum_{i=1}^{n}\theta_{i1}$ is then close to $\mu$, i.e. the surplus $s_1(\theta)$ from providing the public good is positive with probability close to one. The probability of a veto is also close to one, however: with probability close to one there are individuals with $\theta_{i1} < 1$. This observation raises the question how $\mathbb{P}_{all}$ behaves if both $m$ and $n$ grow at the same time.

**Proposition 3.** *Suppose that $n = \gamma\, m$ for $\gamma > 0$. Then* $\lim_{m\to\infty} \mathbb{P}_{all}(m) > 0$.

5 The result of Mailath and Postlewaite applies to any admissible mechanism. Therefore it applies, in particular, to the all-or-nothing-mechanism.

6 The assumption of log-concavity is satisfied by many well-known probability distributions, including the uniform distribution, the normal distribution or the logistic distribution, see Bagnoli and Bergstrom (2005).

The argument in the proof of Proposition 2 is easily adapted to deal with $m$ and $n$ growing at the same rate. The conclusion is weaker in that case, $\mathbb{P}_{all}(m)$ is bounded from below by a positive constant that may be smaller than 1.[7] The fact that it is bounded away from zero implies that the impossibility result that is obtained for $m = 1$ does not extend to this case.

## 4. On the optimality of the all-or-nothing-mechanism

We will now show that, under certain conditions, the all-or-noting-mechanism is a second-best mechanism, i.e. a mechanism which maximizes the expected surplus

$$E\left[\frac{1}{n}\sum_{i=1}^{n} u_i(\theta)\right] = E\left[\sum_{k=1}^{m} s_k(\theta)\, q_k(\theta)\right]$$

over the set of mechanisms which satisfy the constraints in (1)-(3).

In doing so, we will treat $n$ as fixed. As a consequence, the all-or-nothing-mechanism is not a first-best mechanism.[8] For any good $k$, the probability of the event $s_k(\theta) < 0$ is strictly positive. As a consequence, the all-outcome includes projects with a negative surplus with positive probability. Moreover, for large $m$, this probability is close to one.

The following assumption greatly simplifies the proof that the all-or-nothing mechanism is a second-best mechanism. We further discuss its role below.

**Assumption 1.** There is a fixed order for the implementation of projects. Specifically, $q_l(\theta) = 1$ implies $q_k(\theta) = 1$, for all $k \leq l$.

The assumption means that there is a natural order in which public projects can be undertaken. Project 2 can be undertaken only after project 1 has been implemented, project 3 can be implemented only after project 2 has been implemented and so on. The set of possible public good outcomes therefore becomes smaller. Specifically, the possible outcomes can be represented by the set $K' = \{0, 1, \ldots, m\}$ where outcome $k' \in K'$ indicates that all public goods with an index smaller or equal $k'$ are provided. The role that this assumption plays in our proof will become clear. It ensures that all logically conceivable preferences over the set of outcomes can be represented by an additively separable utility function, i.e. we can satisfy a universal domain requirement without having to introduce utility functions that allow for substitutes or complements in public goods preferences.

**Theorem 1.** *Suppose $f$ is symmetric around its mean, log-concave and has full support on an interval $[\mu - a, \mu + a]$, for some $a > 0$. Also suppose that Assumption 1 holds. Then, the all-or-nothing-mechanism is a second-best mechanism.*

In the following, we first explain the key steps in the proof of the theorem, with formal details relegated to the Appendix. We then provide a discussion of Assumption 1.

[7] In the Appendix, we also show that $\mathbb{P}_{all}(m) \to 1$ if $m$ and $n$ do not grow at the same rate, but $\frac{m}{n} \to \infty$.

[8] As $n \to \infty$, for any $k$, $\frac{1}{n}\sum_{i=1}^{n} \theta_{ik}$ converges in probability to $\mu > 1$. Hence, the all-outcome converges in probability to a first best outcome.

### 4.1. Proof of Theorem 1

The following lemma implies that, in what follows, we can limit attention to mechanisms that involve equal cost sharing.

**Lemma 1.** *If a direct mechanism is anonymous and satisfies the incentive constraints in (2) and the budget constraints in (3) then, for all $i$ and for all $\theta$, $t_i(\theta) = \sum_{k=1}^{m} q_k(\theta)$.*

The lemma and its proof in part A.5 of the Appendix are of independent interest. It is useful for the same reason as the characterization of incentive compatibility via the envelope theorem in Bayesian mechanism design. This characterization yields, for instance, the well-known revenue equivalence result in auction theory. Knowing what individual payments have to look like makes it possible to focus on allocation rules, as opposed to allocation and payment rules. This greatly simplifies the analysis. Here, however, the argument involves not only incentive constraints, but the interplay of incentive constraints, budget constraints and the requirement of anonymity.[9] Also note that Lemma 1 holds irrespectively of whether or not Assumption 1 is satisfied.

By Lemma 1 and Assumption 1 individual $i$'s preferences over the outcomes $k' \in K'$ of the mechanism can be represented by the utility function

$$\hat{u}_i(\theta) = \sum_{l=1}^{k'} (\theta_{il} - 1) \, . \tag{4}$$

According to the impossibility result by Gibbard (1973) and Satterthwaite (1975), with a universal domain of preferences, any incentive compatible mechanism that has more than two outcomes is dictatorial and therefore violates the requirement of anonymity. By the following lemma, under Assumption 1, all logically conceivable rankings over the set of outcomes can be represented by utility functions that take the form in (4); i.e. the universal domain property is satisfied.

**Lemma 2.** *Let $\mathcal{R}$ be the set of preference relations over the set of outcomes $K'$. To every $\succ_i \in \mathcal{R}$ there exists a type $\theta_i \in \Theta_i$ so that, for any $k, k' \in K'$, $k' \succ_i k$ if and only if*

$$\sum_{l=1}^{k'} (\theta_{il} - 1) > \sum_{l=1}^{k} (\theta_{il} - 1) \, .$$

**Corollary 1.** *Under Assumption 1, admissible mechanisms have at most two outcomes.*

The only way in which we can satisfy the individuals' participation constraints is to have the status quo as one of these two outcomes. Thus, the specification of the alternative to the status quo is only one degree of freedom that is left; i.e. the class of admissible mechanisms is of the form *nothing or all public goods with an index below $k'$*. Let $S(k')$ be the expected surplus that is generated by such a mechanism. By the following Lemma, the surplus is strictly increases in this index, with the implication that the *all-or-nothing-mechanism* is the optimal mechanism.

9 The Lemma relates to previous results in the literature. Kuzmics and Steg (2017) treat the case $m = 1$ and do not impose the requirement of anonymity. With this requirement, their analysis also yields equal cost sharing. Bierbrauer and Hellwig (2016), again for $m = 1$, invoke an additional requirement of coalition-proofness in their proof that every admissible mechanism involves equal cost sharing. Our analysis shows that the requirement of coalition-proofness is not needed to obtain equal cost sharing.

**Lemma 3.** *Suppose $f$ is symmetric around its mean, log-concave and has full support on an interval $[\mu - a, \mu + a]$, for some $a > 0$. Then, for any $k'$, $S(k') < S(k' + 1)$.*

### 4.2. *On Assumption 1*

The universal domain property is needed to justify our use of the Gibbard and Satterthwaite theorem. Assumption 1 ensures that we can satisfy this property by focusing on a simple class of utility functions, $\hat{u}_i(\theta) = \sum_{l=1}^{k'}(\theta_{il} - 1)$. In the Appendix, we present an example that illustrates that, without this Assumption, there are preference profiles that cannot be represented by an additively separable utility function. If we do not impose Assumption 1, we have to consider a richer class of preferences to satisfy the universal domain property. Once such preferences are allowed for, we can again appeal to the Gibbard and Satterthwaite theorem and focus on mechanisms with at most two outcomes. Hence, Assumption 1 allows us to apply the Gibbard and Satterthwaite theorem in a setting with additive separable utility functions which makes the analysis tractable.

## 5. Concluding remarks

We have shown that bundling many public goods facilitates public goods provision in the presence of incentive and participation constraints. Additional public goods come with additional resource requirements. Thus, sufficient state capacity is necessary to reap the benefits from bundling. If bundling is not an option, as Mailath and Postlewaite (1990) have shown, it is impossible to have positive provision levels - unless the government uses its coercive power to collect contributions from individuals who do not value the public good. This also points to a potential drawback of deciding about every public project on a stand-alone-basis. If the benefits from bundling remain unused, there will be an underprovision of public goods if participation constraints are respected, or, if they are not respected, public goods provision will be unnecessarily controversial as it will create winners and losers.

We show that the all-or-noting-mechanism is an optimal mechanism. This mechanism stipulates equal cost sharing and an exhaustion of the state's capacity to provide public goods, unless there is an individual who prefers to stick to the status quo. Obviously, the optimal mechanism can therefore be implemented by a simple voting procedure. Everybody can vote "yes" or "no" and the capacity to provide public goods is exhausted unless there is an individual who votes "no." This observation links this paper with the literature that analyzes welfare-maximizing voting procedures from a mechanism design perspective.[10]

## Appendix A. Proofs

### A.1. *Proof of Proposition 1*

Before we prove Proposition 1 we state Corollary 2.1 in Proschan (1965)[11]: Let $f$ be log-concave and symmetric around 0, i.e. $f(x) = f(-x)$ for all $x$. Also suppose that $f(x) > 0$,

[10] The seminal contribution to this research program is Rae (1969); recent contributions include Schmitz and Tröger (2012), Azrieli and Kim (2014), Drexl and Kleiner (2018) or Bierbrauer and Hellwig (2016).

[11] Proschan refers to distributions with a log-concave density as *Polya frequency functions of order 2*.

whenever $x \in [-a, a]$ and $f(x) = 0$, otherwise. Let $(X_k)_{k=1}^m$ be a collection of *i.i.d.* random variables with density $f$. Then for any $t \in (0, a)$ the probability of the event $\frac{1}{m}\sum_{k=1}^m X_k \leq t$ is strictly increasing in $m$.

Note that because of symmetry, the probability of the event $\frac{1}{m}\sum_{k=1}^m X_k \leq t$ is equal to the probability of the event $\frac{1}{m}\sum_{k=1}^m X_k \geq -t$. The probability of the event $\frac{1}{m}\sum_{k=1}^m X_k < -t$ is therefore strictly decreasing in $m$.

**Proof of Proposition 1.** We seek to show that the probability of the event $\exists i \in I\colon \frac{1}{m}\sum_{k=1}^m \theta_{ik} < 1$ is smaller than the probability of the event $\exists i \in I\colon \frac{1}{m+1}\sum_{k=1}^{m+1} \theta_{ik} < 1$. Since preferences are $iid$, this holds if and only if, for any given individual $i$, the probability of $\frac{1}{m}\sum_{k=1}^m \theta_{ik} < 1$ is smaller than the probability of $\frac{1}{m+1}\sum_{k=1}^{m+1} \theta_{ik} < 1$. As an implication of Corollary 2.1 in Proschan (1965), the probability of an event $\frac{1}{m}\sum_{k=1}^m \theta_{ik} < x$, where $x < \mu$ is strictly decreasing in $m$. To see this note that $\frac{1}{m}\sum_{k=1}^m \theta_{ik} < x$ holds if and only if $\frac{1}{m}\sum_{k=1}^m \theta_{ik} - \mu < x - \mu$. The right hand side of this inequality is a negative number, the left hand side is an average of random variables with mean zero. By Corollary 2.1. in Proschan (1965) the probability of this event strictly decreasing in $m$. The proposition follows from this fact upon setting $x = 1$.

### A.2. *Proof of Proposition 2*

We start by bounding the probability that any individual vetoes against the all-outcome with Chebychev's inequality

$$\mathbb{P}\left[\frac{1}{m}\sum_{k=1}^{m}\theta_{ik} < 1\right] \leq \mathbb{P}\left[\left|\mu - \frac{1}{m}\sum_{k=1}^{m}\theta_{ik}\right| > \mu - 1\right] \leq \frac{\sigma^2}{(\mu-1)^2}\frac{1}{m}. \tag{5}$$

Using Inequality (5) we bound the probability that no one vetoes from below

$$\begin{aligned}\mathbb{P}_{all}(m) &= \mathbb{P}[\text{no individual vetoes the provision of } m \text{ public projects}]\\ &= (1 - \mathbb{P}[\text{i vetoes the provision of } m \text{ public projects}])^n\\ &\geq \left(1 - \tfrac{\sigma^2}{(\mu-1)^2}\tfrac{1}{m}\right)^n .\end{aligned}$$

The lower bound goes to 1 as $m \to \infty$. Hence, $\lim_{m\to\infty} \mathbb{P}_{all}(m) = 1$.

### A.3. *Proof of Proposition 3*

We adapt the argument in the proof of Proposition 2. There it was shown that $\mathbb{P}_{all}(m) \geq (1 - \frac{d}{m})^{\gamma\, m}$ for $d = \frac{\sigma^2}{(\mu-1)^2}$. The right hand side of this inequality converges to $e^{-\gamma\, d} > 0$ as $m \to \infty$.

### A.4. *Limit probability as $\frac{m}{n}$ becomes unbounded*

Define $h(m) = \frac{n}{m}$. We seek to show that $\mathbb{P}_{all}(m)$ converges to 1 as $h(m)$ converges to 0. We again adapt the argument in the proof of Proposition 2 where it was shown that $\mathbb{P}_{all}(m) \geq$

$\left(1-\frac{d}{m}\right)^{h(m)\,m}$ for $d=\frac{\sigma^2}{(\mu-1)^2}$. Since $\left(1-\frac{d}{m}\right)^m$ is an increasing function of $m$, it is, for all $m$, (weakly) larger than $1-d$. Hence,

$$\mathbb{P}_{all}(m) \geq (1-d)^{h(m)}\,.$$

The right hand side of this inequality goes to 1 as $h(m) \to 0$.

### A.5. Proof of Lemma 1

We occasionally use $q(\theta)$ as a shorthand for $(q_k(\theta))_{k\in K}$. Moreover, we will use $v(\theta_i, q(\theta))$ as a shorthand for $\sum_{k\in K}\theta_{ik}\, q_k(\theta)$. For a given state $\theta$, we write $K_0(\theta)=\{k \mid q_k(\theta)=0\}$ for the set of projects that are not implemented and, analogously, $K_1(\theta)=\{k \mid q_k(\theta)=1\}$ for the complementary set. Also, for any $k$, we write $\underline{\theta}_k(\theta)=\min_{i\in I}\ \theta_{ik}$ and $\overline{\theta}_k(\theta)=\max_{i\in I}\ \theta_{ik}$. If this creates no confusion, we will occasionally suppress the dependence of these minima and maxima on the state $\theta$ and simply write $\underline{\theta}_k$ and $\overline{\theta}_k$. The following lemma will also prove useful.

**Lemma A.1.** *Consider two states $\theta$ and $\theta'$ such that the following holds:*

i) $\theta'_{-i}=\theta_{-i}$,
ii) $\theta'_{ik}>\theta_{ik}$ *for all $k$ with* $q_k(\theta)=1$,
iii) $\theta'_{ik}<\theta_{ik}$ *for all $k$ with* $q_k(\theta)=0$.

*Then, for all $k$, $q_k(\theta')=q_k(\theta)$ and $t_i(\theta)=t_i(\theta')$.*

**Proof.** The incentive constraints for individual $i$ in state $\theta'$ imply

$$t_i(\theta)-t_i(\theta') \geq v(\theta'_i, q(\theta)) - v(\theta'_i, q(\theta'))\,. \tag{6}$$

Note that

$$\begin{aligned}
v(\theta'_i, q(\theta)) - v(\theta'_i, q(\theta')) &= \textstyle\sum_{k\in K}\theta'_{ik}(q_k(\theta)-q_k(\theta')) \\
&= \textstyle\sum_{k\in K}\theta_{ik}(q_k(\theta)-q_k(\theta')) \\
&\quad + \textstyle\sum_{k\in K_1(\theta)}(\theta'_{ik}-\theta_{ik})(1-q_k(\theta')) \\
&\quad + \textstyle\sum_{k\in K_0(\theta)}(\theta'_{ik}-\theta_{ik})(0-q_k(\theta')) \\
&\geq v(\theta_i, q(\theta)) - v(\theta_i, q(\theta'))\,.
\end{aligned}$$

The inequality follows because, by property ii), $(\theta'_{ik}-\theta_{ik})(1-q_k(\theta'))\geq 0$ for $k\in K_1(\theta)$, and by property iii), $(\theta'_{ik}-\theta_{ik})(0-q_k(\theta'))\geq 0$ for $k\in K_0(\theta)$. Moreover,

$$v(\theta'_i, q(\theta)) - v(\theta'_i, q(\theta')) > v(\theta_i, q(\theta)) - v(\theta_i, q(\theta'))\,, \tag{7}$$

if there is $k\in K_1(\theta)$ with $q_k(\theta')=0$ or $k\in K_0(\theta)$ with $q_k(\theta')=1$. Suppose in the following that this is the case. Then, inequalities (6) and (7) imply that

$$t_i(\theta)-t_i(\theta') > v(\theta_i, q(\theta)) - v(\theta_i, q(\theta'))\,.$$

Hence, a violation of incentive compatibility for individual $i$ in state $\theta$. Thus, the assumption that there is $k\in K_1(\theta)$ with $q_k(\theta')=0$ or $k\in K_0(\theta)$ with $q_k(\theta')=1$ has led to a contradiction and must be false. Hence, for all $k$, $q_k(\theta) = q_k(\theta')$. It remains to be shown that $t_i(\theta) = t_i(\theta')$. With $q(\theta) = q(\theta')$, (6) becomes

$$t_i(\theta) - t_i(\theta') \geq 0\,. \tag{8}$$

Analogously, the incentive constraint $t_i(\theta) - t_i(\theta') \leq v(\theta_i, q(\theta)) - v(\theta_i, q(\theta'))$ becomes

$$t_i(\theta) - t_i(\theta') \leq 0\,. \tag{9}$$

Inequalities (8) and (9) imply $t_i(\theta) = t_i(\theta')$. □

**Proof of Lemma 1.** Consider a state $\theta$ and suppose that there exist individuals $i$ and $j$ with $t_i(\theta) \neq t_j(\theta)$. We show that this leads to a contradiction to the assumption that the given mechanism is anonymous, incentive compatible and satisfies ex post budget balance. Assume without loss of generality that $t_i(\theta) > \sum_{k \in K} q_k(\theta)$. We construct state $\theta'$ so that[12]

i) $\theta'_{-i} = \theta_{-i}$,
ii) $\theta'_{ik} = \overline{\theta}_k$ for all $k \in K_1(\theta)$,
iii) $\theta'_{ik} = \underline{\theta}_k$ for all $k \in K_0(\theta)$.

By Lemma A.1, $q(\theta) = q(\theta')$ and $t_i(\theta) = t_i(\theta')$. Therefore, $t_i(\theta') > \sum_{k \in K} q_k(\theta')$ and there must exist an individual $j$ with $t_j(\theta') < \sum_{k \in K} q_k(\theta')$. Otherwise there would be a budget surplus in state $\theta'$.

We now construct state $\theta''$ so that

i) $\theta''_{-j} = \theta'_{-j}$,
ii) $\theta''_{jk} = \overline{\theta}_k$ for all $k \in K_1(\theta)$,
iii) $\theta''_{jk} = \underline{\theta}_k$ for all $k \in K_0(\theta)$.

Again, by Lemma A.1, $q(\theta') = q(\theta'')$ and $t_j(\theta') = t_j(\theta'')$. Also, by anonymity, $t_i(\theta'') = t_j(\theta'')$. Since $t_i(\theta'') < \sum_{k \in K} q_k(\theta'')$ there must exist $h \neq i, j$ with $t_h(\theta'') > \sum_{k \in K} q_k(\theta'')$. Otherwise there would be budget deficit.

We now repeat this construction until we have a state $\theta^{(n)}$ so that all individuals have the same type, i.e. so that for all $i \in I$, $\theta^{(n)}_{ik} = \overline{\theta}_k$ for all $k \in K_1(\theta)$ and $\theta^{(n)}_{ik} = \underline{\theta}_k$ for all $k \in K_0(\theta)$. By anonymity $t_i(\theta^{(n)}) = t_j(\theta^{(n)})$, for all $i$ and $j$. By the arguments above, we either have $t_i(\theta^{(n)}) > \sum_{k \in K} q_k(\theta^{(n)})$ or $t_i(\theta^{(n)}) < \sum_{k \in K} q_k(\theta^{(n)})$ in this state, a contradiction to budget balance.

### *A.6. Proof of Lemma 2*

Given a preference relation $\succ_i$ over $K'$ denote by $r(\succ_i, k)$ the rank of alternative $k$. Hence, $k' \succ_i k$ if and only if $r(\succ_i, k') < r(\succ_i, k)$. To construct the corresponding type $\theta_i$, we let $\theta_{ik} = d(\succ_i, k) + 1$ where $d(\succ_i, k)$ is the rank difference of two neighboring alternatives, $d(\succ_i, k) = r(\succ_i, k-1) - r(\succ_i, k)$. We now show that $r(\succ_i, k') < r(\succ_i, k)$ if and only if $\sum_{l=1}^{k'} (\theta_{il} - 1) >$

[12] The reasoning that follows also applies if $\theta = \theta'$.

$\sum_{l=1}^{k}(\theta_{il}-1)$. To see that this is the case, suppose that $k' > k$ (the case $k' < k$ is analogous) and note that, by construction,

$$
\begin{aligned}
&\textstyle\sum_{l=1}^{k'}(\theta_{il}-1) > \sum_{l=1}^{k}(\theta_{il}-1) && \Longleftrightarrow \\
&\textstyle\sum_{l=k+1}^{k'}\theta_{il} > k'-k && \Longleftrightarrow \\
&\textstyle\sum_{l=k+1}^{k'}(d(\succ_i,l)+1) > k'-k && \Longleftrightarrow \\
&\textstyle\sum_{l=k+1}^{k'}d(\succ_i,l) > 0 && \Longleftrightarrow \\
&\textstyle\sum_{l=k+1}^{k'}r(\succ_i,l-1)-r(\succ_i,l) > 0 && \Longleftrightarrow \\
&r(\succ_i,k) > r(\succ_i,k')\,.
\end{aligned}
$$

### A.7. Proof of Lemma 3

Denote by $p_{yes}(k')$ the probability that any one individual $i$ opts for public goods provision – i.e. the probability of the event $\sum_{l=1}^{k'}(\theta_{il}-1)\geq 0$ – under a *nothing or all public goods with an index below $k'$ mechanism*. From the arguments in the proof of Proposition 1,

$$p_{yes}(k') < p_{yes}(k'+1)\,. \tag{10}$$

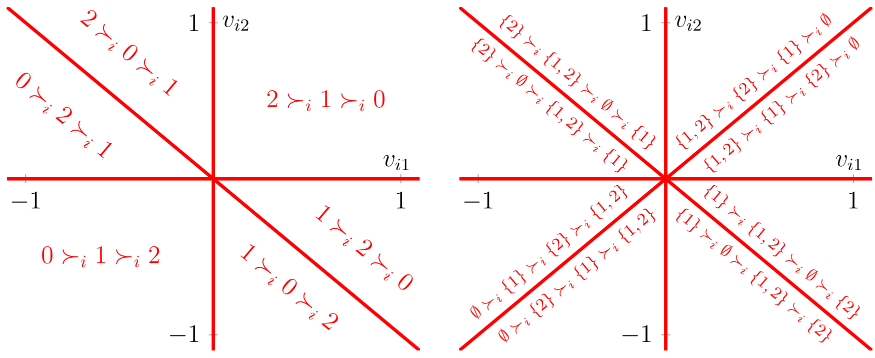
Also note that

$$
\begin{aligned}
S(k'+1) &= E\left[\frac{1}{n}\sum_{i=1}^{n}\sum_{l=1}^{k'+1}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'+1}(\theta_{il}-1)\geq 0 \quad\text{and}\quad \forall j\neq i,\ \sum_{l=1}^{k'+1}(\theta_{jl}-1)\geq 0\right)\right] \\
&= p_{yes}(k'+1)^{n-1}\frac{1}{n}\sum_{i=1}^{n}E\left[\sum_{l=1}^{k'+1}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'+1}(\theta_{il}-1)\geq 0\right)\right],
\end{aligned}
\tag{11}
$$

where $\mathbf{1}$ is the indicator function. Moreover,

$$
\begin{aligned}
&E\left[\sum_{l=1}^{k'+1}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'+1}(\theta_{il}-1)\geq 0\right)\right] \\
&\geq E\left[\sum_{l=1}^{k'+1}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'+1}(\theta_{il}-1)\geq 0 \quad\text{and}\quad \sum_{l=1}^{k'}(\theta_{il}-1)\geq 0\right)\right] \\
&\geq E\left[\sum_{l=1}^{k'+1}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'}(\theta_{il}-1)\geq 0\right)\right].
\end{aligned}
\tag{12}
$$

The first inequality holds because the second expression looks at a smaller set of events among those that satisfy $\sum_{l=1}^{k'+1}(\theta_{il}-1)\geq 0$. The second inequality holds because the sum in the third expression is now both over events with $\sum_{l=1}^{k'+1}(\theta_{il}-1)\geq 0$ and over events with $\sum_{l=1}^{k'+1}(\theta_{il}-1)<0$, among those that satisfy $\sum_{l=1}^{k'}(\theta_{il}-1)\geq 0$. We now rewrite this last expression as



(a) Assumption 1 (b) No restriction

Fig. 1. Example.

$$\begin{aligned}
&E\left[\sum_{l=1}^{k'+1}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'}(\theta_{il}-1)\geq 0\right)\right]\\
&=p_{yes}(k')E[\theta_{ik'+1}-1]+E\left[\sum_{l=1}^{k'}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'}(\theta_{il}-1)\geq 0\right)\right]\\
&=p_{yes}(k')(\mu-1)+E\left[\sum_{l=1}^{k'}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'}(\theta_{il}-1)\geq 0\right)\right]\\
&>E\left[\sum_{l=1}^{k'}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'}(\theta_{il}-1)\geq 0\right)\right].
\end{aligned} \tag{13}$$

Equation (11) and the inequalities (10), (12) and (13) imply

$$S(k'+1) \quad > \quad p_{yes}(k')^{n-1}\frac{1}{n}\sum_{i=1}^{n}E\left[\sum_{l=1}^{k'}(\theta_{il}-1)\mathbf{1}\left(\sum_{l=1}^{k'}(\theta_{il}-1)\geq 0\right)\right] \quad = \quad S(k')\,.$$

### *A.8. An example illustrating Assumption 1*

Consider the case with two public projects that is illustrated in Fig. 1. For ease of exposition, the figure shows valuations net of the per capita provision costs, $v_{ik}=\theta_{ik}-1$. With Assumption 1 there are three outcomes: no provision, provision of good 1, and the provision of goods 1 and 2. The number of possible preference orderings over these three outcomes equals $3!=6$. Fig. 1(a) illustrates that for every preference ordering $\succ_i$ there is some type $\theta_i$ that induces it. For instance, valuations in the upper right quadrant give rise to the following ranking: providing two public goods is preferred over providing one public good. Providing one public good in turn is preferred over no provision at all. As the Figure shows, any one of the 6 possible preference profiles corresponds to some region in Fig. 1(a). Without Assumption 1, a fourth outcome comes into play, namely to provide the public good with index $k=2$, but not to provide the public good with index 1. There are now $4!=24$ preference orderings over these outcomes. As Fig. 1(b) shows, only eight of these preference relations can be represented in the given type space. For example, a preference relation so that $\{2\}\succ_i\{1\}\succ_i\emptyset\succ_i\{1,2\}$ is incompatible with it.

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